ABSTRACT
Shake tables are used for studying the effects of vibration or evaluating the physical properties of materials or structures. In this paper, cross-talk phenomenon in a two-axis electro-dynamic shake table utilizing hydrostatic oil bearings is examined by both experimental and multi-body dynamics (MBD) approaches. Once cross-talk begins, the performance of the shake table significantly degrades. In this investigation of cross-talk phenomena, first, experiments were conducted and then simulation analyses were carried out. The authors improvised and approximated an oil bearing element to use with a general purpose MBD software. This paper describes the modeling of the oil bearing, the mechanical system and the electrical system. The results of experimental studies using an actual shake table and computer simulations were compared. Acceleration responses are in fairly good agreement under specific conditions. Mechanisms of the cross-talk were partly elucidated in terms of natural frequencies and mode shapes with respect to the change of the vertical location of the horizontally directed actuating force or the change of the oil film thickness. These studies also proved the usefulness of using MBD for analyzing the coupled mechanical and electrical systems. Due to the lack of a hydrostatic oil bearing element in RecurDyn at the time of this study, the authors employed nonlinear springs to simulate the hydrostatic behavior of the oil bearings.

1. INTRODUCTION
Vibration testing machines, also called shake tables are used for studying the effects of vibration and evaluating the physical properties of materials or structures, Cyril, et al. (1961), Taniguchi (1985), Nagamatsu (1993). The force causing motion of the table is produced electro-dynamically by the interaction between a current flow in the driver coil and an intense magnetic field which cuts the coil. A two-axis small or medium sized electro-dynamic shake table is comprised of a table attached to hydrostatic oil bearings. The hydrostatic oil bearings are placed beneath and at the side of the table. Each oil-bearing allows large unidirectional movements of the table, tangential to each bearing surface. Two driver coils which are attached to the table structure via the oil bearings, and located in the air-gap fluxes, experience electro-dynamically generated forces which are proportional to the magnitude of electric current flowing in their wires.

Many home appliances in need of vibration control such as washing machines and vacuum cleaners, and also car components as well as many other industrial products are used daily by millions of people. For such items, shake tables are necessary for conducting vibration tests in order to develop these high performance products. Two-axis shake tables are often used to experimentally verify vibration-proof performance of newly developed mechanical products.

There are several types of shake tables, including hydraulic-servo, electric-servo, and electro-dynamic types. In all types of shake tables, there is a common difficulty in their operation, i.e. the cross-talk phenomenon. Unlike in a single axis shake table, the generation of the cross-talk in a two-axis shake table is inevitable. This is because the two-axis shake table is one which moves in a two dimensional plane and contains two actuators, which are in perpendicular planes to each other. One-axis motion of the table interacts and interferes with the other axis motion during operation. This interference is called the cross-talk phenomenon. Often, in the design and manufacture of two-axis shake tables, one of the most important concerns is the reduction of this cross-talk. This is achieved by properly arranging parts and components of the shake table from a structural view point. Then, the design advances to the next
step of how to reduce the cross-talk by means of advanced control technologies for the whole shake table system.

In our study, we focused on the dynamics of cross-talk of the mechanical system of a shake table which involves electric circuits for actuating the table without advanced controllers. To investigate the cross-talk phenomenon, the authors conducted experiments and MBD simulations for a two dimensional two-axis electro-dynamic shake table with hydro-static oil bearings as the supporting parts. Papers on cross-talk studies of shake tables are few, but there are some studies on upgrading the control of shake table performance in order to reduce interaction, a kind of cross-talk, between the shake table and the super-structures, Nagai, et. al. (2000), Shimizu, et. al. (2004), Stoten, et. al. (2005).

2. EXPERIMENTS OF TWO-AXIS SHAKE TABLE

Ideally, a three dimensional three-axis shake table would be used for investigating cross-talk phenomenon of shake tables. In order to simplify the problem, the experiments were restricted to two dimensional two-axis in planar motions. Thus in this paper, the authors explain a two dimensional two-axis shake table. This study on cross-talk will begin with a basic setup of the table without a load and further studies will progress to higher stages. The first stage of this investigation will be on cross-talk phenomenon for a fundamental shake table set-up without any loads on the table. The experimental set-up (included a balancing weight) is shown in Figure 1 with sensor locations. The shake table consists of a table with balance weight, two oil bearings, two actuators for horizontal and vertical directions and a supporting structure. The authors conducted experiments under several conditions, namely, the table without loads, the table without loads but with balance weight and the table with loads. In this paper, the authors restrict themselves to explaining tests for the model without loads but with a balance weight of 34 kg. Figure 2 shows the acceleration responses in the horizontal direction at the center position and in the vertical direction at the edge position of the non loaded-table for horizontal actuation.

3. ANALYSIS METHOD AND COMPUTER SOFTWARE

An MBD approach was employed to numerically investigate the performance of an electro-dynamic two-axis shake table. The general purpose computer software RecurDyn by FunctionBay Inc. (2009), which is a fast, efficient and easy to use multi-body dynamics analysis software, is used. RecurDyn automaticallyformulates recursive equations of motion and constraint equations of mechanical systems. These are called DAE (differential-algebraic equations)

\[
\begin{bmatrix}
M & C_q^T \\
C_q & 0
\end{bmatrix}
\begin{bmatrix}
\dot{q} \\
\lambda
\end{bmatrix}
= \begin{bmatrix}
f^d \\
\gamma
\end{bmatrix}
\]  

(1)

and are based on the inputs of the system geometries and connections. Where \(M\) is the system mass matrix, \(C_q\) the constraint Jacobian, \(q\) the generalized coordinate vector, \(\lambda\) the Lagrange multiplier vector, \(f^d\) the external force vector and \(\gamma\) the vector of acceleration equations of constraints. RecurDyn can also handle electric, control and mechanical coupled systems. In this study, RecurDyn evaluates the mechanical and electrical coupled systems. The Colink toolkit, available in RecurDyn, simulates the mechanical and electric circuit dynamics. Colink, the control system simulator integrated with RecurDyn, gives descriptions using block diagrams similar to that of
MATLAB/Simulink. Since no hydrostatic bearing toolkit was available for our study, the authors developed a pseudo hydrostatic bearing element by means of a nonlinear spring model. This will be explained in detail in section 4.1 (b).

4. MODELING OF ANALYSIS SYSTEM
4.1 Modeling of Mechanical System

A mechanical, electro-dynamic shake table system is analyzed and explained as follows.

(a) Composition of Mechanical System

Figure 3 shows the composition of the shake table which is analyzed in the next chapter (see also Figure 1). The electro-dynamic two-axis shake table is composed of a table, two hydrostatic bearings, horizontal and vertical actuators (with actuating coils) and a supporting structure. The force causing motion of the table is produced electro-dynamically by the interaction between a current flowing in the driver coil and the intense magnetic field which cuts the coil. The table is supported by hydrostatic oil bearings in the horizontal and vertical directions. The horizontal and rotational motions are measured by two acceleration sensors which are installed at the center of the surface and the edge of the table. The weight of the coils, the base hydrostatic bearing and the table is supported by rubber mounts attached to the supporting structure on a reinforced concrete floor. In this modeling for program inputs, all parts (bodies) of the analysis model are assumed to be rigid except for rubber mounts and oil bearings.

(b) Hydrostatic Oil Bearing

Two hydrostatic oil bearings are used in the two-axis shake table to support and transmit the load and to isolate the perpendicular motion with respect to the loading direction. Two driver coils which are attached to the table via the hydrostatic oil bearings, experience electro-dynamically generated forces via electric current. Since the RecurDyn library does not have the element for simulating hydrostatic oil bearings, the authors established a substitute function by use of a nonlinear spring model which can approximately simulate the dynamic behavior of the hydrostatic oil bearing. The load of the hydrostatic cylindrical bearing in Figure 4 can be described by Eq. (2) with respect to the oil film thickness $h$, Sasaki (1958), Yabe (1972), Konami (1999).

$$ W = \frac{A_c P_s}{1 + K_B h^3 / K_c} \quad (2) $$

where, $K_B$, $K_C$ and $A_c$ are given as

$$ K_B = \frac{\pi}{6 \log_e \left(\frac{r_o}{r_i}\right)} \quad K_C = \frac{\pi r_c^4}{8 \ell} \quad A_c = \frac{\pi \left(r_c^2 - r_i^2\right)}{2 \log_e \left(\frac{r_o}{r_i}\right)} $$

Other variables are defined as $W$: load [N], $P_s$: hydraulic pressure [N/m$^2$], $h$: oil film thickness [m], $r_o$: cylinder outer radius [m], $r_i$: cylinder inner radius [m], $r_c$: radius of oil canal [m] and $\ell$: length of oil canal [m]. The load capacity, Eq.(2), which is a function of the oil film thickness, was described using a nonlinear spring force as a bushing element in RecurDyn for the hydrostatic bearing. The bushing element in RecurDyn is an element that defines independent spring constants and damping coefficients in the translation and rotation directions. The restoring force in the direction of compression is given by
the curve in Figure 5. No restoring force is given in the direction perpendicular to the direction of compression. Weak damping forces were given in both translational and rotational directions to suppress unstable computational errors.

Figure 5. LOAD CAPACITY AGAINST OIL FILM THICKNESS.

The oil film thicknesses of $h_u$ and $h_l$ in the initial state are called the initial oil film thicknesses, $h_u0$ and $h_l0$, ($h_u0 = h_l0 = h_0$), where $h_u$ and $h_l$ denote upper and lower oil film thickness, respectively, as shown in Figure 6. In our computational analysis, we set the oil film thickness to $h_0 = 0.045\text{mm}$ for the initial model which we will denote as the basic model in Table 1.

4.2 Electric circuit system

The exciting force of an electro-dynamic shake table is described by Fleming’s left-hand rule as

$$\vec{F} = I \times \vec{B} \quad (4)$$

where $\vec{F}$ is a force vector, $\times$ the outer product of two vectors, $I$ the current flowing vector and $\vec{B}$ the magnetic flux density. In an electro-dynamic shake table, the driving current flowing into the coil generates the exciting force written in Eq. (4) which results in actuating the shake table. The signal flow diagram of the shake table with the flowing current in the driver coils is shown in Figure 7.

Eqs. (5) to (7) describe the input voltage and it’s relation to the exciting force.

$$i_d = A_m e_d \quad (5)$$

$$i_d = \frac{Ri_d}{L} - \frac{\Gamma x_c}{L} + \frac{e_d}{L} \quad (6)$$

$$f_d = \Gamma i_d \quad (7)$$

where $\Gamma = Bl$, $B$, $l$, $L$, $R$, $i_d$ and $x_c$ indicate the power coefficient $(\text{T} \cdot \text{m})$, the magnetic flux density $(\text{T})$, length of coil $(\text{m})$, effective inductance of the driver coil $(\text{H})$, effective resistance of the driver coil $(\Omega)$, driving current $(\text{A})$ and velocity of the driver coil $(\text{m/s})$, respectively.

4.3 Modeling Details of the Coupled System

To compute the coupling analysis for the total system composed of the mechanical and electric systems, the block diagram of Eqs. (5) to (7) was built by Colink in RecurDyn and is shown in Fig.8. In the following discussion, fundamental characteristics of mechanical and electric systems subjected to sinusoidal waves will be examined without the presence of any performance enhancing controllers.
The driving force was divided equally into eight parts per driver coil in the circumferential direction, and was applied to the body representing the driver coil as shown in Figure 9.

5. PARAMETER STUDIES AND THEIR CONSIDERATIONS

The purpose of this simulation study is to consider the dynamic behavior of the shake table by comparing the simulation results with the experiment results under the condition of applying the exciting force in the horizontal direction only.

5.1 Modeling of Mechanical System (Initial Model)

The initial parameters of the analysis model were determined by the manufacture’s specifications of the shake table. Some of those values are different from the actual ones. In order to check the sensitivity of parameters affecting the behavior of the shake table without loads, the authors calculated the change of responses due to the change of inductances. The changes in the acceleration responses for horizontal motion at the center of the table and for the vertical motion at the edge of the table (see sensor locations in Figure 1) due to the change of electric inductance and resistance parameters were studied. The frequency response function for the initial model was calculated by RecurDyn-Colink, and the first peak frequency of the horizontal acceleration of the table was different from the experimental one. In order for the calculated peak frequency to agree well with the experimental one, the values of inductance and resistance had to be modified. Figure 10 shows the frequency responses of this model subjected to sinusoidal force excitation with the amplitude of 1328N ($e_d = 0.02V$) and Figure 11 shows the displacement responses of the mode shape at the horizontal peak frequency (60Hz). The authors define this model as the basic model in the parameter study.

In Figure 10, it is observed that the vertical peak frequency at the table edge of the basic model is 90Hz which is much higher than that of the actual shaking table (50Hz) and the vertical peak acceleration of the basic model is almost half of the actual one. Figure 11 shows maximum displacement responses of typical points on the shake table. When the horizontal displacement becomes positive the vertical displacement also becomes positive, thus the center of rotation of the shake table is located above the table. Since the first natural frequency in the horizontal direction that the authors obtained in the basic model coincided with that of the actual model, focus was placed on the vertical peak frequency and its peak accelerations. In an effort to find the reasons why the
Several trial computations suggested the reasons why the differences occurred. There may be two main reasons. One reason is due to the difference of the vertical locations of the actuating force in the horizontal direction and the other is due to the difference of oil film thickness between the computational model and the actual table. The authors investigated qualitative and quantitative differences between the analysis models and the actual table by modifying model parameters from the basic model. Table 1 shows the simulation cases of computational models which specify the points of interest.

Table 1. SIMULATION CASES INVOLVING COMPUTATIONAL CONDITIONS

<table>
<thead>
<tr>
<th>Case</th>
<th>Models</th>
<th>Computational conditions (points of interest)</th>
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</thead>
<tbody>
<tr>
<td>case 1</td>
<td>Basic model</td>
<td>initial model for parameter simulation</td>
</tr>
<tr>
<td>case 2</td>
<td>ALL model</td>
<td>the modified model by changing Actuating Location Level</td>
</tr>
<tr>
<td>case 3</td>
<td>OFT model</td>
<td>the modified model by changing Oil Film Thickness</td>
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5.2 Change of Responses by Change of Actuating Location

The vertical location of the actuating force in the horizontal direction was modified by changing the location of the supporting rubber mounts, which support the weight of the table and the hydrostatic oil bearings underneath the table, 5 mm downward. Consequently, the analysis model (case 2) in which the vertical location of the horizontally directed excitation was shifted 5 mm upward. Figure 12 shows the computed frequency response functions of the vertical acceleration (○ mark) at the edge of the table and compares the computational results with the experimental ones (Δ mark). Figure 13 shows displacement responses at typical points of the shake table and the mode shape at the peak frequency in the model.

The change in frequency response can be observed by the comparison of Figure 10 and Figure 12. The amount of the acceleration of the table of the ALL model (case 2) increased compared to that of the basic model. It was also found that the peak acceleration in the vertical direction approaches the experimental one at the edge of the table, but the peak frequency does not change. Thus, it is learned that changing the vertical actuating location level is not an essential countermeasure to solving this problem, because this countermeasure cannot control the natural frequency of the system.

5.3 Change of Responses by Change of Oil Film Thickness

In order to control the natural frequencies of the system, mass and/or stiffness elements needed to be changed. In our analysis, spring elements, i.e. the vertical spring rigidities of the bearings which support the table, needed to be modified. Vertical spring rigidities are determined by the oil film thickness of hydrostatic oil bearings, so the oil film thickness in the hydrostatic bearing...
was adjusted. Figure 14 shows the acceleration frequency response function of the table at the center (horizontal direction) and the edge (vertical direction) for the OFT model (the model whose oil film thickness is raised to 60 μm from the basic model, 45 μm). Natural frequency of the vertical direction at the edge of the table decreased to 60 Hz and neared the actual one, 50 Hz. At the same time, the peak response also increased and neared the experimental one. Figure 15 shows the displacement responses of the mode shape at the peak frequency of 60 Hz for horizontal excitations. Figures 10 and 14 shows that the peak frequency was lowered. Figures 11 and 15 show that the rotation angle and displacement increased. This was due to the lowering of the rotational rigidity of the system caused by the increase of oil film thickness. Thus we can confirm that the analysis results in the OFT model differed from the basic model and approached the experimental results for both horizontal and vertical directions at two sensor locations.

5.4 Comparison of Results and Considerations

In the preceding sections, three types of models for the shake table were numerically investigated: the basic model (case 1), the AFL model (case 2, the modified actuating force location model) and the OFT model (case 3, the modified oil film thickness model). In this section, these results are compared and considered in terms of frequency responses and peak frequencies of the models.

(1) Frequency response functions

As expected from Figs. 10, 12 and 14, the maximum accelerations of the calculated and measured results at the center of the table surface in the horizontal direction for the basic model (case 1), the AFL model (case 2) and the OFT model (case 3) are almost identical. In the vertical direction, the calculated results of the OFT model (Fig. 14) are much closer to the experimental results than those of the basic model (Fig. 10) or the AFL model (Fig. 12). From these observations, the vertical location of the actuating force in the basic model seems to be different from the actual shake table. As previously explained in section 5.3, the maximum responses of the OFT model are much closer to that of the basic model. From this observation, the oil film thickness of the actual shake table seems to be larger than that of the basic model.

(2) Peak frequencies and their response amplitudes

Table 2 lists the peak frequencies which, are also considered to be the natural frequencies, of the shake table system. Almost no differences between the basic, the ALL and the OFT models in the horizontal motion are observed. It is believed that neither, changing the vertical location level of the actuating force nor changing the thickness of the oil film strongly affects the natural frequency of the shake table in the horizontal direction. On the other hand, changing the thickness of oil film strongly affects the rotational rigidity of the hydrostatic bearing and consequently the natural frequency of the shake table system. This can be observed by the comparison of the peak frequencies of the basic model (90 Hz) and the OFT model (60 Hz) in the vertical direction. Still, the value of 60 Hz for the OFT model is different from the value of 50 Hz.

Figure 14. FREQUENCY RESPONSE OF OFT MODEL (CASE 3 MODEL).

Figure 15. MODE RESPONSES OF OFT MODEL AT 60 Hz (CASE 3 MODEL).
in the actual shake table. From this observation, the OFT model appears to be stiffer than the actual table. Table 3 lists the peak acceleration amplitudes in the horizontal direction at the center of the table and the vertical direction at the edge of the table subjected to the horizontal force of 1328N. Table 3 shows that the responses of the OFT model are fairly close to those of the actual shake table in the horizontal and vertical directions.

<table>
<thead>
<tr>
<th>Table 2: PEAK FREQUENCIES OF EACH MODEL.</th>
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<tr>
<td>Model</td>
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<tr>
<td>-------------</td>
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<tr>
<td>actual table</td>
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<tr>
<td>basic model</td>
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<tr>
<td>AFL model</td>
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<td>OFT model</td>
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<table>
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<th>Table 3: PEAK ACCELERATIONS OF EACH MODEL.</th>
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<td>Model</td>
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<td>OFT model</td>
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6. CONCLUSIONS AND FUTURE PROBLEMS

At the start of this study the authors thought that the main factors for governing crosstalk phenomena in electrodynamic shake tables supported by the hydrostatic oil bearings would be:
(1) the vertical location level of actuating force in horizontal direction, and
(2) the oil film thickness in hydrostatic oil bearing.

Multi-body dynamics simulation studies were carried out considering the above conditions, using the basic, the AFL and the OFT models. The basic model was established as an initial model. From this model, the two modified models were formed. In order to perform parameter studies, firstly a nonlinear spring element for approximating the characteristics of the hydrostatic oil film bearing was established and used for simulation. From the simulation studies of the three models given in the preceding chapters, the authors obtained the following conclusions;
(a) Hydrostatic oil film bearing could be successfully modeled by the nonlinear spring,
(b) If the vertical location of the actuating force in the horizontal direction changes, the amount of rotation of the table changes due to the change of the equivalently applied torque about the mass center. If there is no change in the spring constants in this model, the peak frequency doesn't change, because the change of applied torque does not affect the stiffness or mass of the system.
(c) Since the rigidities of the vertical and rotational directions change due to the changes of oil film thickness, the natural frequencies of the shake table also change.

From the above studies, the cross-talk phenomena were partly elucidated. The next step is to conduct more precise research by incorporating flexible multibody dynamics into further studies of this problem. The above studies also show that MBD is very useful for analyzing mechanical-electrical coupled systems.

REFERENCES